Experiment No **:** M4

Name of Experiment: Two Dimensional Collisions

Objective: To examine conservation of momentum and energy for elastic collision in two dimensions. To demonstrate experimentally that momentum is a vector and energy is a scaler quantity.

Keywords: Velocity, kinetic energy, conservative force, conservation of energy, conservation of momentum, vector-scalar quantities

Theoretical Information:

Conservation of Mechanical Energy

Energy is a scalar quantity. It can be divided into two forms, kinetic and potential. In the following equations, potential energy and kinetic energy are shown with *U* and *K*, respectively. Before the collision is represented by "initial", and after collision is represented by "final" subscripts. According to the work-energy theorem, the change in kinetic energy of an object is equal to work that has done by the net force acting on the object.

$$W = K_{final} - K_{initial} \tag{4.1}$$

Let's assume that only conservative forces are generating the work. The work done by conservative forces is independent of the path and equal to the negative of the potential energy change. When these are combined together,

$$W = K_{final} - K_{initial} = -U_{final} - U_{initial} \tag{4.2}$$

is obtained. When this last equation can be written as,

$$K_{final} + U_{final} = K_{initial} + U_{initial} \tag{4.3}$$

is obtained. Here, the left side of the equation represents the total energy after the collision whereas the right side of the equation represents the total energy before the collision. So, as $E = K + U$;

$$E_{final} = E_{initial} \tag{4.4}$$

is obtained. If only conservative forces are generating the work, the mechanic energy of the system is also conserved, thus it is independent of time. The equation 4.4 represents the mechanic energy conservation in 1,2 or 3 dimensions. This equation shows the relationship between the velocity and position of the object. During the movement of the object, both kinetic and potential energy will change but their sum will be constant. The law of conservation of the energy states that in an insulated system total energy of the system is conserved. It is possible to write this statement in form of,

$$\frac{dE}{dt} = 0 \Rightarrow E \equiv constant \tag{4.5}$$

Conservation of Momentum

Momentum $\vec{P}$, of an object with mass *m* and velocity $\vec{v}$ is defined by the multiplication of mass and velocity vector.

$$\vec{P} = m\vec{v} \tag{4.6}$$

Momentum is a vector quantity. Newton's Second Law can be written as,

$$\sum \vec{F} = m\vec{a} = m\frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt} \tag{4.7}$$

Let us consider of a system with zero net force acting on it.

$$\sum \vec{F} = 0 = \frac{d\vec{P}}{dt} \qquad 4.8$$

That means the change of momentum over time is zero, i.e. momentum is time independent. In other words, the momentum of the system is conserved.

$$\vec{P}_{initial} = \vec{P}_{final} \qquad 4.9$$

Let's assume only one component of force, for examples that F_y is zero. Then, let's write the Newton's Law in form of components:

$$F_x = \frac{dP_x}{dt}; F_y = \frac{dP_y}{dt} = 0; F_z = \frac{dP_z}{dt} \qquad 4.10$$

As it can be seen, the solution of the first and the third equation is time independent. However the derivation of P_y *is zero, in other words* P_y *is constant.* Thus y-component of the momentum is conserved

In a closed system of particles, i.e. no external force acting on the system and particles only interact with each other, total momentum of the system is conserved. (See Problem 1). Here the total momentum of the system and the vector sum of the momentums should be understood.
N particle system of masses m_1 , m_2 ,....... m_N can be generalized according to the statement above. Total momentum of such system composed of particles in a given time can be written as:

$$\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 + \ldots\ldots + \vec{P}_N \qquad 4.11$$

Here $\vec{P}_1 = m_1\vec{v}_1$,$\vec{P}_2 = m_2\vec{v}_2, \ldots$ can be written.
The summation in the equation 2.11 is a vector summation. In this case, when equation 4.3 generalized;

$$\sum \vec{F}_{ext} = \frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\left(\vec{P}_1 + \vec{P}_2 + \ldots\ldots + \vec{P}_N\right) \qquad 4.12$$

will be obtained.
Here $\vec{F}_{ext}$, defines the net external force acting on the system of particles. In other words, the external force is different from interaction force of the particles between them. These external forces can be friction or gravitation. Thus if there is no external force acting on the system of particles, the total momentum of the system will be conserved. So;

$$\frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\left(\vec{P}_1 + \vec{P}_2 + \ldots\ldots + \vec{P}_N\right) = 0 \Rightarrow \vec{P}_{tot} = \vec{P}_1 + \vec{P}_2 + \ldots\ldots + \vec{P}_N \equiv constant \qquad 4.13$$

The equation above is also a vector quantity. In a system of particles with no net force (or in an isolated system) , total momentum of the system in any given time will be the same.
Quantities that does not change under certain conditions such as energy and momentum are defined as *constants of the motion.* These constants simplify the solution of motion equations.
Momentum and energy conservation expressions of elastic collision of two particles in a plane are defined as follows;

Momentum-horizantal: $m_1\vec{v}_{1x} + m_2\vec{v}_{2x} = m_1\vec{v}'_{1x} + m_2\vec{v}'_{2x}$ 4.14

Momentum-vertical: $m_1\vec{v}_{1y} + m_2\vec{v}_{2y} = m_1\vec{v}'_{1y} + m_2\vec{v}'_{2y}$ 4.15

Kinetic energy : $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2$ 4.16

Center of mass (CM) is the other quantity that is compared and investigated in this experiment. Center of mass of a uniform cube, cylinder, sphere and other symmetrical objects is its geometric center. Center of mass of the two objects of the same mass will be located at the midpoint of the line that connects their geometric centers. However, if one of the objects is heavier than the other, then the center of mas will shift towards the heavier one.

The mass should be redefined for various mass distributions. The position vector, $\vec{R}$ of system of N partical with position vectors $\vec{r}_{1,}\vec{r}_{2,}$....., $\vec{r}_N$ and masses m_1, m_2,....,m_N is defined as in equation 4.17. Here, $\vec{r}$ vectors are position vectors of each particle in the coordinate system whereas $\vec{R}$ represents the position vector of the center of mass.

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_N\vec{r}_N}{m_1 + m_2 + \ldots + m_3} \qquad 4.17$$

If the position of particles changes with time, position of the center of mass also changes the vector changing ratio of center of mas can be considered as the velocity of (CM).

$$\vec{v}_{CM} = \frac{d\vec{R}}{dt} \qquad 4.18$$

If the derivative of both sides of the equation 2.17 will be taken for constant mass particles;

$$\dot{\vec{R}} = \frac{m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2 + \ldots + m_N\dot{\vec{r}}_N}{m_1 + m_2 + \ldots + m_N} \Rightarrow \vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_N\vec{v}_N}{m_1 + m_2 + \ldots + m_N} \qquad 4.19$$

Equation (4.19) will be obtained. The dots on of the equation 4.19 are annotation of derivative with respect to time which is the velocity of the mass. When these derived equations are adapted to our two-disc system in the experiment;

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \qquad 4.22$$

and the masses in the equation 4.22 simplified because the mass of the discs are equal ($m_1=m_2=m$); equation 4.23 will be obtained.

$$\vec{R} = \frac{m\vec{r}_1 + m\vec{r}_2}{m + m} \Rightarrow \vec{R} = \frac{m(\vec{r}_1 + \vec{r}_2)}{2m} \Rightarrow \vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2} \qquad 4.23$$

In that case; if the derivative of position vectors with respect to time will be taken, the velocity of the CM will be:

$$\vec{V}_{CM} = \frac{\vec{v}_1 + \vec{v}_2}{2} \qquad 4.24$$

Equation 4.24 has important outcomes. It means that while momentum is conserved, the velocity of CM is constant (constant velocity, no change in magnitude and direction.). Thus the center of mass of an isolated system with conserved momentum always moves with constant velocity in linear motion. Thus of our two-disc system should be $\overrightarrow{V}_{\mathrm{CM}}^{\mathrm{initial}} = \vec{V}_{\mathrm{CM}}^{\mathrm{final}}$ for before and after the collision.